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LIGHT REGIME OF A TWO-LAYER ATMOSPHERE-SEA MEDIUM

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LIGHT REGIME OF A TWO-LAYER ATMOSPHERE-SEA MEDIUM

S. D. Gutshabash

The light regimes of the sea and the atmosphere, as well as the intensity of diffuse radiation at the sea-atmosphere boundary, may be determined from the following assumption. The atmosphere, with an optical thickness of τ_1 and a quantum survival probability λ_1, is adjacent to the sea, whose optical thickness is infinite and whose quantum survival probability is λ_2. In a calculation of the atmospheric light regime, the fact is taken into consideration that the atmosphere is irradiated by both parallel solar rays, and radiation which is diffusely reflected from the sea. The light regime of the sea may be calculated under the assumption that the sea is irradiated not only by solar rays, which are attenuated by the atmosphere, but also by diffuse radiation of the atmosphere. /44* /45

At first, for purposes of simplicity, the scattering of light in the sea and the atmosphere is assumed to be isotropic, and the boundary is not considered. Then the solution obtained is generalized to the case of nonisotropic scattering. In conclusion, the orthotropic boundary between the atmosphere and the sea is considered.

Isotropic Scattering

Light regime of the atmosphere. Let us assume that an isotropically scattering atmosphere is irradiated by parallel rays, whose flux through a unit area perpendicular to the rays

*Numbers in the margin indicate pagination of original foreign text.

at the level of the upper boundary equals πS, and the angle of incidence is $i = \arccos \zeta$. The source function $B_0(\tau, \zeta)$ is determined by the following equation [1]

$$B_0(\tau, \zeta)=\frac{\lambda_1}{2} \int_{-1}^{1} I(\tau, \eta, \zeta)\, d\eta+\frac{\lambda_1}{4} S e^{-\frac{\tau}{\zeta}}, \tag{1}$$

where $I(\tau, \eta, \zeta)$ is the strength of diffuse radiation at the optical depth τ, $\vartheta = \arccos \eta$ — angle with the normal. If, in addition, the atmosphere is irradiated by diffuse radiation of the strength $I_2(\tau, \eta, \zeta)$ coming from the sea, then the source function is

$$B(\tau, \zeta)=\frac{\lambda_1}{2} \int_{-1}^{1} I(\tau, \eta, \zeta)\, d\eta+\frac{\lambda_1}{4} S e^{-\frac{\tau}{\zeta}}+ \\ +\frac{\lambda_1}{2} \int_{0}^{1} I_2(\tau_1, \eta, \zeta)\, e^{-\frac{\tau_1-\tau}{\eta}}\, d\eta. \tag{2}$$

Equation (2) differs from (1) by a free term. However, the free term of Equation (2) is the superposition of terms of the same type as the free term of Equation (1). In view of the linearity of both equations, this means that the solution of Equation (2) is a superposition of the solution of Equation (1). Consequently, we may write

$$B(\tau, \zeta)=B_0(\tau, \zeta)+\frac{2}{S} \int_{0}^{1} I_2(\tau_1, \eta', \zeta)\, B_0(\tau_1-\tau, \eta')\, d\eta'. \tag{3}$$

Thus, if the source function $B_0(\tau, \zeta)$ is known, for the case when the atmosphere is irradiated by parallel rays, we may use integration to find the source function for the case when the atmosphere is irradiated by diffuse radiation of the sea, in addition. /46

The strength of diffuse radiation at the optical depth τ, which goes in the upwards direction (Figure 1), is determined by the formula

$$I(\tau, -\eta, \zeta)=\int_{\tau}^{\tau_1} B(\tau', \zeta) e^{-\frac{\tau'-\tau}{\eta}} \frac{d\tau'}{\eta} \tag{4}$$

and in the downward direction

$$I(\tau, \eta, \zeta)=\int_{0}^{\tau} B(\tau', \zeta) e^{-\frac{\tau-\tau'}{\eta}} \frac{d\tau'}{\eta}. \tag{5}$$

Instead of B(τ, ζ), substituting its value from Equation (3), we obtain

$$\begin{aligned} I(\tau, -\eta, \zeta)&=I_0(\tau, -\eta, \zeta)+\frac{2}{S}\int_0^1 I_2(\tau, \eta', \zeta) I_0(\tau_1-\tau_1, \eta, \eta')\, d\eta' + \\ &+I_2(\tau_1, \eta, \zeta) e^{-\frac{\tau_1-\tau}{\eta}}; \\ I(\tau, \eta, \zeta)&=I_0(\tau, \eta, \zeta)+\frac{2}{S}\int_0^1 I_2(\tau_1, \eta', \zeta) I_0(\tau_1-\tau, -\eta, \eta')\, d\eta', \end{aligned} \tag{6}$$

where

$$\begin{aligned} I_0(\tau, -\eta, \zeta)&=\int_{\tau}^{\tau_1} B_0(\tau', \zeta) e^{-\frac{\tau'-\tau}{\eta}} \frac{d\tau'}{\eta}; \\ I_0(\tau, \eta, \zeta)&=\int_0^{\tau} B_0(\tau', \zeta) e^{-\frac{\tau-\tau'}{\eta}} \frac{d\tau'}{\eta}. \end{aligned} \tag{7}$$

Formulas (6) determine the light regime of the atmosphere. If we set τ = 0 in the first of them, we obtain the strength of the radiation $I_3(0, -\eta, \zeta)$ which is diffusely reflected by the two-layer atmosphere-sea medium. As is customary, assuming

$$I_0(0, -\eta, \zeta)=S\rho_1(\tau_1, \eta, \zeta)\zeta; \quad I_0(\tau_1, \eta, \zeta)=S\sigma_1(\tau_1, \eta, \zeta)\zeta, \tag{8}$$

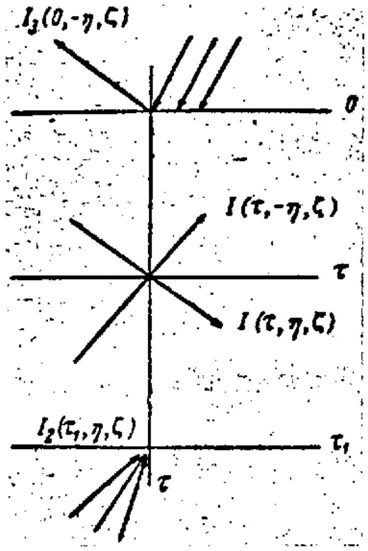

Figure 1. Atmosphere irradiated by solar rays and diffuse radiation of the sea.

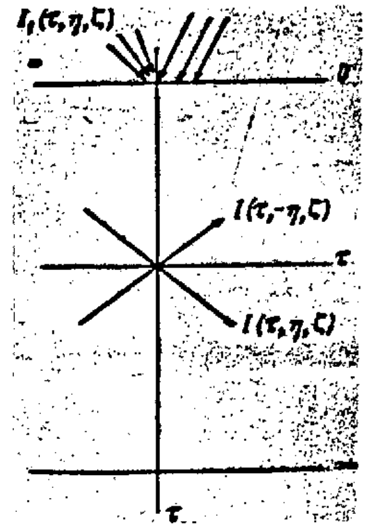

Figure 2. Sea irradiated by solar rays attenuated by the atmosphere, and diffuse radiation of the atmosphere.

we obtain

$$I_3(0, -\eta, \zeta) = S\rho_1(\tau_1, \eta, \zeta)\zeta + 2\int_0^1 I_2(\tau_1, \eta', \zeta)\sigma_1(\tau_1, \eta, \eta')\eta' \, d\eta' + I_2(\tau_1, \eta, \zeta)e^{-\frac{\tau_1}{\eta}}. \quad (9)$$

The last term is added to allow for radiation passing directly from the surface of the sea.

Light regime of the sea. We shall assume that the optical thickness $\tau = 0$ corresponds to the surface of the sea. The flux /47 of solar rays through a unit surface at the sea level is designated by $\pi S'$. Apparently we have

$$\pi S' = \pi S e^{-\frac{\tau_1}{\zeta}} \quad (10)$$

If it assumed that the sea is irradiated only by parallel solar rays, then for the source function $\bar{B}_0(\tau, \zeta)$ we may write

$$\overline{B}_0(\tau, \zeta) = \frac{\lambda_2}{2} \int_{-1}^{1} I(\tau, \eta, \zeta)\, d\eta + \frac{\lambda_2}{4} S' e^{-\frac{\tau}{\zeta}}. \tag{11}$$

Taking into account the diffuse radiation of the atmosphere, we obtain

$$\begin{aligned}\overline{B}(\tau, \zeta) = \frac{\lambda_2}{2} \int_{-1}^{1} I(\tau, \eta, \zeta)\, d\eta + \frac{\lambda_2}{4} S' e^{-\frac{\tau}{\zeta}} + \frac{\lambda_2}{2} \int_{0}^{1} \times \\ \times I_1(\tau_1, \eta', \zeta)\, e^{-\frac{\tau}{\eta'}}\, d\eta'.\end{aligned} \tag{12}$$

Utilizing, just as above, the principle of superposition, we obtain

$$\overline{B}(\tau, \zeta) = \overline{B}_0(\tau, \zeta) + \frac{2}{S'} \int_{0}^{1} I_1(\tau_1, \eta', \zeta)\, \overline{B}_0(\tau, \eta')\, d\eta'. \tag{13}$$

The strength of the diffuse radiation at the optical depth τ (Figure 2) passing upward is /48

$$I(\tau, -\eta, \zeta) = \int_{\tau}^{\infty} \overline{B}(\tau', \zeta)\, e^{-\frac{\tau' - \tau}{\eta}} \frac{d\tau'}{\eta}$$

and in the depths of the sea

$$I(\tau, \eta, \zeta) = \int_{0}^{\tau} \overline{B}(\tau', \zeta)\, e^{-\frac{\tau - \tau'}{\eta}} \frac{d\tau'}{\eta}. \tag{14}$$

Utilizing Expression (13), we have

$$\begin{aligned} I(\tau, -\eta, \zeta) &= \overline{I}_0(\tau, -\eta, \zeta) + \frac{2}{S'} \int_{0}^{1} I_1(\tau_1, \eta', \zeta)\, \overline{I}_0(\tau, -\eta, \eta')\, d\eta', \\ I(\tau, \eta, \zeta) &= \overline{I}_0(\tau, \eta, \zeta) + \frac{2}{S'} \int_{0}^{1} I_1(\tau_1, \eta', \zeta)\, \overline{I}_0(\tau, \eta, \eta')\, d\eta' + \\ &\quad + I_1(\tau_1, \eta, \zeta)\, e^{-\frac{\tau}{\eta}}. \end{aligned} \tag{15}$$

Here

$$\bar{I}_0(\tau, -\eta, \zeta) = \int_\tau^\infty \bar{B}_0(\tau', \zeta) e^{-\frac{\tau'-\tau}{\eta}} \frac{d\tau'}{\eta};$$
$$\bar{I}_0(\tau, \eta, \zeta) = \int_0^\tau \bar{B}_0(\tau', \zeta) e^{-\frac{\tau-\tau'}{\eta}} \frac{d\tau'}{\eta}. \qquad (16)$$

To determine the strength at the atmosphere-sea boundary, we assume $\tau = \tau_1$ in the second of the equations (6). Utilizing Formulas (8), we find the strength of the diffuse radiation passing from the atmosphere to the sea $I_1(\tau_1, \eta, \zeta)$. Similarly, assuming $\tau = 0$ in the first of the equations (15), we find the strength of the radiation which is diffusely reflected by the sea. As a result, we obtain a system of equations for the unknown strengths

$$I_1(\tau_1, \eta, \zeta) = S\sigma_1(\tau_1, \eta, \zeta)\zeta + 2\int_0^1 I_2(\tau_1, \eta', \zeta)\rho_1(\tau_1, \eta, \eta')\eta'\, d\eta';$$
$$I_2(\tau_1, \eta, \zeta) = S\rho(\eta, \zeta) e^{-\frac{\tau_1}{\zeta}}\zeta + 2\int_0^1 I_1(\tau_1, \eta', \zeta)\rho(\eta, \eta')\eta'\, d\eta'. \qquad (17)$$

In the second of these equations, instead of the strength, we have introduced the reflection coefficient $\rho(\eta, \zeta)$ by means of the relationship

$$\bar{I}_0(0, -\eta, \zeta) = S\rho(\eta, \zeta)\zeta. \qquad (18)$$

The system, which is similar to system (17), was examined /49
previously [2]. The solution may be found by means of certain auxiliary functions which only depend on one of the arguments η or ζ.

Nonisotropic Scattering

Let us extend the results obtained to the case of nonisotropic scattering in the atmosphere and the sea.

Light regime of the atmosphere. If the atmosphere is irradiated by parallel solar rays, then the source function is

$$B_0(\tau, \eta, \zeta, \varphi, \varphi_0)=\frac{\lambda_1}{4\pi}\int_0^{2\pi} d\varphi' \int_{-1}^{1} I(\tau, \eta', \zeta, \varphi', \varphi_0)\times \times x_1(\eta, \eta', \varphi, \varphi')\,d\eta'+\frac{\lambda_1}{4} S x_1(\eta, \zeta, \varphi, \varphi_0)\,e^{-\frac{\tau}{\zeta}}, \tag{19}$$

where $x(\eta, \zeta, \varphi, \varphi_0)$ is the scattering indicatrix of the atmosphere, φ and φ_0 are the azimuths calculated from a certain direction.

If we assume that the atmosphere is irradiated by diffuse radiation of the sea, then

$$B(\tau, \eta, \zeta, \varphi, \varphi_0)=B_0(\tau, \eta, \zeta, \varphi, \varphi_0)+ +\frac{\lambda_1}{4\pi}\int_0^{2\pi} d\varphi' \int_0^1 I_2(\tau_1, \eta', \zeta, \varphi', \varphi_0)\,x_1(\eta, \eta', \varphi, \varphi')\,e^{-\frac{\tau_1-\tau}{\eta'}}\,d\eta'. \tag{20}$$

Utilizing the principle of superposition, we obtain

$$B(\tau, \eta, \zeta, \varphi, \varphi_0)=B_0(\tau, \eta, \zeta, \varphi, \varphi_0)+ +\frac{1}{\pi S}\int_0^{2\pi} d\varphi' \int_0^1 I_2(\tau_1, \eta', \zeta, \varphi', \varphi_0)\,B_0(\tau, -\tau, \eta, \eta', \varphi, \varphi')\,d\eta'. \tag{21}$$

where $I_2(\tau_1, \eta, \zeta, \varphi, \varphi_0)$ is the strength of radiation which is diffusely reflected by the sea. Substituting (21) in Equations (4) and (5), $\boxed{\text{we}}$ find the light regime at the optical depth τ

$$I(\tau, -\eta, \zeta, \varphi, \varphi_0) = I_0(\tau, -\eta, \zeta, \varphi, \varphi_0) + \\ + \frac{1}{\pi S} \int_0^{2\pi} d\varphi' \int_0^1 I_2(\tau_1, \eta', \zeta, \varphi', \varphi_0) I_0(\tau_1 - \tau, \eta, \eta', \varphi, \varphi') d\eta'; \\ I(\tau, \eta, \zeta, \varphi, \varphi_0) = I_0(\tau, \eta, \zeta, \varphi, \varphi_0) + \\ + \frac{1}{\pi S} \int_0^{2\pi} d\varphi' \int_0^1 I_2(\tau_1, \eta', \zeta, \varphi', \varphi_0) I_0(\tau_1 - \tau, -\eta, \eta', \varphi, \varphi') d\eta'. \qquad (22)$$

Formulas (22) determine the light regime of the atmosphere in the case when the atmosphere is irradiated both by parallel rays, and by diffuse radiation of the sea. /50

Light regime of the sea. If the sea is irradiated by parallel solar rays attenuated by the atmosphere, then the source function at the optical depth τ is

$$\overline{B}_0(\tau, \eta, \zeta, \varphi, \varphi_0) = \frac{\lambda_2}{4\pi} \int_0^{2\pi} d\varphi' \int_{-1}^1 I(\tau, \eta', \zeta, \varphi', \varphi_0) x_2(\eta, \eta', \varphi, \varphi') \times \\ \times d\eta' + \frac{\lambda_2}{4} S' x_2(\eta, \zeta, \varphi, \varphi_0) e^{-\frac{\tau}{\zeta}}, \qquad (23)$$

where $x_2(\eta, \zeta, \varphi, \varphi_0)$ is the scattering indacatrix of the sea. Taking into account diffuse radiation of the atmosphere, we shall have

$$\overline{B}(\tau, \eta, \zeta, \varphi, \varphi_0) = \overline{B}_0(\tau, \eta, \zeta, \varphi, \varphi_0) + \\ + \frac{1}{4\pi} \int_0^{2\pi} d\varphi' \int_0^1 I_1(\tau, \eta', \zeta, \varphi', \varphi_0) x_2(\eta, \eta', \varphi, \varphi') e^{-\frac{\tau}{\eta'}} d\eta', \qquad (24)$$

or

$$\overline{B}(\tau, \eta, \zeta, \varphi, \varphi_0) = \overline{B}_0(\tau, \eta, \zeta, \varphi, \varphi_0) + \\ + \frac{1}{\pi S'} \int_0^{2\pi} d\varphi' \int_0^1 I_1(\tau, \eta', \zeta, \varphi', \varphi_0) \overline{B}_0(\tau, \eta, \eta', \varphi, \varphi') d\eta'. \qquad (25)$$

Utilizing Formulas (14), we find the strength of diffuse radiation at the optical depth τ — light regime of the sea

$$
\begin{gathered}
\bar{I}(\tau, -\eta, \zeta, \varphi, \varphi_0)=\bar{I}_0(\tau, -\eta, \zeta, \varphi, \varphi_0)+ \\
+\frac{1}{\pi S'}\int_0^{2\pi} d\varphi' \int_0^1 I_1(\tau_1, \eta', \zeta, \varphi', \varphi_0)\bar{I}_0(\tau, -\eta, \eta', \varphi, \varphi')\,d\eta'; \\
\bar{I}(\tau, \eta, \zeta, \varphi, \varphi_0)=\bar{I}_0(\tau, \eta, \zeta, \varphi, \varphi_0)+\frac{1}{\pi S'}\int_0^{2\pi} d\varphi' \int_0^1 \times \\
\times I_1(\tau_1, \eta', \zeta, \varphi', \varphi_0)\bar{I}_0(\tau, \eta, \eta', \varphi, \varphi')\,d\eta'+ \\
+I_1(\tau_1, \eta, \zeta, \varphi, \varphi_0)\,e^{-\frac{\tau}{\eta}}.
\end{gathered}
\tag{26}
$$

Just as above, assuming $\tau = \tau_1$ in the second of the formulas (22), and $\tau = 0$ in the first of the formulas (26), we obtain a system of equations which determines the strength at the sea-atmosphere boundary

$$
\begin{gathered}
I_1(\tau_1, \eta, \zeta, \varphi, \varphi_0)=S\sigma_1(\tau_1, \eta, \zeta, \varphi, \varphi_0)\zeta+ \\
+\frac{1}{\pi}\int_0^{2\pi} d\varphi' \int_0^1 I_2(\tau_1, \eta', \zeta, \varphi', \varphi_0)\rho_1(\tau_1, \eta, \eta', \varphi, \varphi')\eta'\,d\eta'; \\
I_2(\tau_1, \eta, \zeta, \varphi, \varphi_0)=S\rho(\eta, \zeta, \varphi, \varphi_0)\,e^{-\frac{\tau_1}{\zeta}}\zeta+ \\
+\frac{1}{\pi}\int_0^{2\pi} d\varphi' \int_0^1 I_1(\tau_1, \eta', \zeta, \varphi', \varphi_0)\rho(\eta, \eta', \varphi, \varphi')\eta'\,d\eta'.
\end{gathered}
\tag{27}
$$

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If we represent the scattering indicatrices in terms of Legendre polynomials, then, the solution of this system may be obtained by means of certain auxiliary functions which depend on one of the variables η or ζ. We should note that a system similar to this one was studied previously.

Scattering of Light in the Sea with Allowance for the Orthotropic Boundary between the Atmosphere and the Sea

Let us examine the problem of light scattering in the two-layer atmosphere-sea medium, under the condition that the boundary between the sea and the atmosphere is orthotropic. In certain

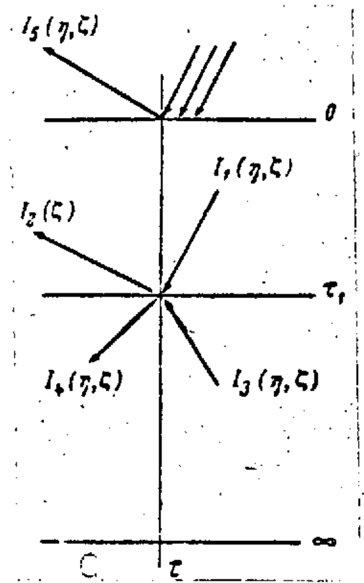

Figure 3. Two-layer medium with orthotropic boundary.

cases which occur in practice (ripples on the sea), this boundary is close to the boundary actually observed. For purposes of simplicity, we shall first assume that the light scattering in both media is isotropic, and then the results obtained will be extended to the case of nonisotropic scattering.

We shall use A to designate the ratio between the flux reflected from the boundary between the media and the incident flux (albedo), in the first medium. In the second medium, we shall use B. Generally speaking, the albedo depends on the angle of incidence for the radiation, but we shall assume that A = const and B = const.

We shall use $I_1(\eta, \zeta)$ to designate the strength of diffuse radiation falling at the angle ϑ = arccos η to the normal, from the first medium at the boundary between the media. Since the boundary is orthotropic, the strength of radiation coming from the boundary $I_2(\zeta)$ does not depend on η. $I_3(\eta, \zeta)$ — the strength of diffuse radiation falling on the second medium at the boundary, and $I_4(\zeta)$ — that of radiation passing from the boundary within the second medium (Figure 3), have similar properties. For the strengths I_1, I_2, I_3, and I_4, we may write /52

$$I_1(\eta, \zeta) = S\sigma_1(\tau_1, \eta, \zeta)\zeta + \frac{I_2(\zeta)}{\pi} 2\pi \int_0^1 \rho_1(\tau_1, \eta, \eta\zeta\eta' d\eta', \qquad (28)$$

$$I_2(\zeta)=A\left[Se^{-\frac{\tau_1}{\zeta}}\zeta+2\pi\int_0^1\frac{I_1(\eta',\zeta)}{\pi}\eta'\,d\eta'\right]+$$
$$+(1-\nu)\,2\pi\int_0^1\frac{I_3(\eta',\zeta)}{\pi}\eta'\,d\eta'; \qquad (29)$$

$$I_3(\eta,\zeta)=\frac{I_4(\zeta)}{\pi}2\pi\int_0^1\rho(\eta,\eta')\,\eta'\,d\eta'; \qquad (30)$$

$$I_4(\zeta)=B2\pi\int_0^1\frac{I_3(\eta',\zeta)}{\pi}\eta'\,d\eta'+$$
$$+(1-A)\left[Se^{-\frac{\tau_1}{\zeta}}\zeta+2\pi\int_0^1\frac{I_1(\eta',\zeta)}{\pi}\eta'\,d\eta'\right]. \qquad (31)$$

In order to solve the system of equations (28) — (31), we set

$$I_1(\eta,\zeta)=S\sigma_1(\tau_1,\eta,\zeta)\zeta;\quad I_2(\zeta)=SR(\zeta)\zeta;$$
$$I_3(\eta,\zeta)=S\rho(\eta,\zeta)\zeta;\quad I_4(\zeta)=SQ(\zeta)\zeta; \qquad (32)$$

$$\nu_1(\eta)=2\int_0^1\rho_1(\tau_1,\eta,\eta')\,\eta'\,d\eta';\quad \nu_2(\eta)=2\int_0^1\rho(\eta,\eta')\,\eta'\,d\eta';$$
$$\mu(\eta)=e^{-\frac{\tau_1}{\eta}}+2\int_0^1\sigma_1(\tau_1,\eta,\eta')\,\eta'\,d\eta';$$
$$C_1=2\int_0^1\nu_1(\eta)\,\eta\,d\eta;\quad C_2=2\int_0^1\nu_2(\eta)\,\eta\,d\eta.$$

In this notation, the system of equations (28) — (31) may be written in the form

$$\begin{aligned}
\bar{\sigma}_1(\tau_1, \eta, \zeta) &= \sigma_1(\tau_1, \eta, \zeta) + R(\zeta)\nu_1(\eta);\\
R(\zeta) &= A[\mu(\zeta) + C_1R(\zeta)] + (1-B)Q(\zeta)C_2;\\
\bar{\rho}(\eta, \zeta) &= Q(\zeta)\nu_2(\eta);\\
Q(\zeta) &= BC_2Q(\zeta) + (1-A)[\mu(\zeta) + C_1R(\zeta)].
\end{aligned} \tag{33}$$

As a result of the solution of this system, we obtain /53

$$\begin{aligned}
R(\zeta) &= \frac{A + C_2D}{1 - AC_1 - BC_2 - C_1C_2D}\mu(\zeta);\\
Q(\zeta) &= \frac{1-A}{1 - AC_1 - BC_2 - C_1C_2D}\mu(\zeta);\\
\bar{\sigma}_1(\tau_1, \eta, \zeta) &= \sigma_1(\tau_1, \eta, \zeta) + \frac{A + C_2D}{1 - AC_1 - BC_2 - C_1C_2D}\mu(\zeta)\nu_1(\eta);\\
\bar{\rho}(\eta, \zeta) &= \frac{1-A}{1 - AC_1 - BC_2 - C_1C_2D}\nu_2(\eta)\mu(\zeta),
\end{aligned} \tag{34}$$

where

$$D = 1 - A - B. \tag{35}$$

Formulas (34) are the desired formulas.

We obtain the following for the strength of reflected radiation, i.e., radiation which is diffusely reflected by a two-layer medium

$$\begin{aligned}
I_S(\eta, \tau) = S\rho_1(\tau_1, \eta, \zeta)\zeta + \frac{I_2(\zeta)}{\pi}2\pi\int_0^1 \sigma_1(\tau_1, \eta, \eta')\eta' d\eta' +\\
+ I_2(\tau)e^{-\frac{\tau_1}{\eta}},
\end{aligned} \tag{36}$$

or, utilizing the notation introduced above,

$$I_S(\eta, \tau) = S\zeta\left[\rho_1(\tau_1, \eta, \zeta) + \frac{A + C_2D}{1 - AC_1 - BC_2 - C_1C_2D}\mu(\eta)\mu(\tau)\right]. \tag{37}$$

We should note that the expression in the brackets is symmetrical with respect to η and ζ, as is required by the principle of reversibility for optical phenomena.

The values of the auxiliary functions $\nu_1(\tau)$, $\nu_2(\tau)$ and $\mu(\tau)$ may be readily found by means of the Ambartsumyan functions and their zero moments [4].

We should note that when $C_2 = 0$, Formulas (32) and (35) change into the formulas obtained by V. V. Sobolev [4] for a medium limited by an orthotropic base.

In order to determine the light regime of the sea, we shall find the source function. Apparently, in this case

$$\overline{B}(\tau, \zeta) = \frac{\lambda_2}{2} \int_{-1}^{1} I(\tau, \eta, \zeta)\, d\eta + \frac{\lambda_2}{2} I_4(\zeta) \int_0^1 e^{-\frac{\tau}{\zeta}}\, d\zeta. \tag{38}$$

Comparing this expression with the source function (11), we /54
find the following for the case when the sea is irradiated by parallel rays

$$\overline{B}(\tau, \zeta) = \frac{I_4(\zeta)}{S'}\, 2 \int_0^1 \overline{B}_0(\tau, \zeta')\, d\zeta'. \tag{39}$$

Substituting Expression (38) in Formulas (14), we may find the light regime of the sea and the strength of diffuse radiation at the boundary. Then, multiplying Equation (39) by

$$e^{-\tau/\eta} \frac{d\tau}{\eta}$$

and integrating between 0 to ∞, we obtain

$$I_3(\eta, \zeta)=\frac{I_4(\zeta)}{S'} 2\int_0^1 I_0(\eta, \zeta')\,d\zeta'. \tag{40}$$

Assuming

$$I_0(\eta, \zeta)=S'\rho(\eta, \zeta)\zeta, \tag{41}$$

we again arrive at Formula (30).

The results obtained above may be readily extended to the case of nonisotropic scattering of light in the atmosphere and the sea. We shall use $x_1(\gamma)$ to designate the scattering indicatrix in the atmosphere, and $x_2(\gamma)$ — in the sea. Just as previously, we shall assume that the brightness coefficients of each of the media, in the absence of the other, are known: $\rho_1(\tau_1, \eta, \zeta, \varphi, \varphi_0)$, $\sigma_1(\tau_1, \eta, \zeta, \varphi, \varphi_0)$, and $\rho(\eta, \zeta, \varphi, \varphi_0)$.

For the strength at the sea-atmosphere boundary $I_1(\eta, \zeta, \varphi, \varphi_0)$, $I_2(\zeta, \varphi_0)$, $I_3(\eta, \zeta, \varphi, \varphi_0)$, and $I_4(\zeta, \varphi_0)$, assuming the boundary is orthotropic, we obtain

$$I_1(\eta, \zeta, \varphi, \varphi_0)=S\sigma_1(\tau_1, \eta, \zeta, \varphi, \varphi_0)\zeta+ + \frac{I_2(\zeta, \varphi_0)}{\pi}\int_0^{2\pi} d\varphi' \int_0^1 \rho_1(\tau_1, \eta, \eta', \varphi, \varphi')\eta'\,d\eta'; \tag{42}$$

$$I_2(\zeta, \varphi_0)=A\left[Se^{-\frac{\tau_1}{\zeta}}+\int_0^{2\pi} d\varphi' \int_0^1 \frac{I_1(\eta', \zeta, \varphi', \varphi_0)}{\pi}\eta'\,d\eta'\right]+ +(1-B)\int_0^{2\pi} d\varphi' \int_0^1 \frac{I_3(\eta', \zeta, \varphi', \varphi_0)}{\pi}\eta'\,d\eta'; \tag{43}$$

$$I_3(\eta,\ \zeta,\ \varphi,\ \varphi_0)=\frac{I_4(\zeta,\ \varphi_0)}{\pi}\int\limits_0^{2\pi} d\varphi' \int\limits_0^1 \rho(\eta,\ \eta',\ \varphi,\ \varphi')\,\eta'\,d\eta';$$
$$I_4(\zeta,\ \varphi_0)=B\int\limits_0^{2\pi} d\varphi' \int\limits_0^1 \frac{I_3(\eta',\ \zeta,\ \varphi',\ \varphi_0)}{\pi}\,\eta'\,d\eta' + \qquad (44)$$

$$+(1-A)\left[Se^{-\frac{\tau_1}{\zeta}}+\int\limits_0^{2\pi} d\varphi' \int\limits_0^1 \frac{I_1(\eta',\ \zeta,\ \varphi',\ \varphi_0)}{\pi}\,\eta'\,d\eta'\right]. \qquad (45)$$

As is customary, we shall assume that the indicatrices may be /55
represented in terms of Legendre polynomials

$$x_1(\gamma)=\sum_{n=0}^{\infty} x_n P_n(\cos\gamma);\quad x_2(\gamma)=\sum_{n=0}^{\infty} y_n P_n(\cos\gamma). \qquad (46)$$

and then the brightness coefficients are determined by the relationships

$$\rho_1(\tau_1,\ \eta,\ \zeta,\ \varphi,\ \varphi_0)=\sum_{n=0}^{\infty} \rho_1^{(n)}(\tau_1,\ \eta,\ \zeta)\cos n(\varphi-\varphi_0);$$
$$\sigma_1(\tau_1,\ \eta,\ \zeta,\ \varphi,\ \varphi_0)=\sum_{n=0}^{\infty} \sigma_1^{(n)}(\tau_1,\ \eta,\ \zeta)\cos n(\varphi-\varphi_0); \qquad (47)$$
$$\rho(\eta,\ \zeta,\ \varphi,\ \varphi_0)=\sum_{n=0}^{\infty} \rho^{(n)}(\eta,\ \zeta)\cos n(\varphi-\varphi_0).$$

If we try to determine I_1 and I_2 in the form

$$I_1(\eta,\ \zeta,\ \varphi,\ \varphi_0)=S\zeta\bar{\sigma}_1(\eta,\ \zeta,\ \varphi,\ \varphi_0)=S\zeta\sum_{n=0}^{\infty} \bar{\sigma}_1^{(n)}(\eta,\ \zeta)\cos n(\varphi-\varphi_0);$$
$$I_3(\eta,\ \zeta,\ \varphi,\ \varphi_0)=S\zeta\bar{\rho}(\eta,\ \zeta,\ \varphi,\ \varphi_0)=S\zeta\sum_{n=0}^{\infty} \bar{\rho}^{(n)}(\eta,\ \zeta)\cos n(\varphi-\varphi_0). \qquad (48)$$

and substitute (47) and (48) in Equations (42) — (45), then only the first terms remain in all the sums as a result of integration over the azimuth. Thus, the solution of Equations (42) — (45)

may be written similarly to the solution of the System (28) — (31). It thus follows that we may replace $\rho_1(\tau_1, \eta, \eta')$, $\rho(\eta, \eta')$, and $\sigma_1(\tau_1, \eta, \eta')$ in the auxiliary functions $\nu_1(\eta)$ and $\nu_2(\eta)$ by $\rho_1^{(0)}(\tau_1, \eta, \eta')$, $\rho_1^{(0)}(\eta, \eta')$, and $\sigma_1^{(0)}(\tau_1, \eta, \eta')$.

To determine the light regime of the sea, we must find the source function. Apparently, in this case

$$\overline{B}(\tau, \eta, \zeta, \varphi, \varphi_0)=\frac{\lambda_2}{4\pi}\int_0^{2\pi} d\varphi' \int_{-1}^{1} I(\tau, \eta', \zeta, \varphi', \varphi_0)\, x_2 \times \\ \times(\eta, \eta', \varphi, \varphi')\, d\eta' + \frac{\lambda_2}{2\pi} I_4(\zeta, \varphi_0)\int_0^{2\pi} d\varphi' \int_0^1 x_2(\eta, \eta', \varphi, \varphi')\, e^{-\frac{\tau}{\eta'}}\, d\eta'. \tag{49}$$

Comparing this equation with Equation (24) and utilizing the principle of superposition, we have

$$\overline{B}(\tau, \eta, \zeta, \varphi, \varphi_0)=\frac{I_4(\zeta, \varphi_0)}{\pi}\int_0^{2\pi} d\varphi' \int_0^1 \overline{B}_0(\tau, \eta, \eta', \varphi, \varphi')\, d\eta'. \tag{50}$$

Thus, in this case, the source function B may be found, if the source function $\overline{B}_0$ is known for a medium irradiated by parallel rays. /56

Thus, if the source function is known for a medium which is irradiated by parallel rays, then finding the source function for a medium irradiated by diffuse radiation reduces to simple integration over the directions. Thus, the light regime of the atmosphere is determined by Formulas (7), if the atmosphere is irradiated only by parallel solar rays, and it is determined by Formulas (6), if in addition the atmosphere is irradiated by diffuse radiation coming from the surface of the sea. In a similar manner, the light regime of the sea is determined by Formulas (16), if the sea is irradiated only by parallel solar rays attenuated by the atmosphere, and it is determined by Formulas (15), if the sea is irradiated by diffuse radiation of the atmosphere.

REFERENCES

1. Ambartsumyan, V. A. Light Scattering by Planetary Atmospheres. Astronomicheskiy zhurnal, Vol. 19, No. 5, 1942.

2. Gutshabash, S. D. Light Scattering in a Two-Layer Atmosphere. Vestnik LGU, No. 1, 1957, p. 1.

3. Gutshabash, S. D. Diffusion of Radiation in a Two-Layer Medium with Allowance for Phenomena at the Boundary between the Media. Izvestiya AN SSSR, Seriya Geofizika, No. 7, 1963.

4. Sobolev, V. V. Perenos izlucheniya v atmosferakh zvezd i planet (Radiation Transport in the Atmospheres of Stars and Planets). Moscow, GTTI, 1956.

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